P181 SEISMIC WAVEFIELD IN CAUSTIC SHADOW

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Abstract

A new method is proposed for numerical computation of elastic wavefields in a shadow zone of caustics. It is a modification of the method called generalized ray tracing (GRT) by (Hanyga and Helle, 1995). The essential feature of GRT consist in application of expressions which are well defined at caustics and expressed in terms of ray tracing (travel times and the ray amplitudes calculated along coalescing rays). In caustic shadows GRT requires a complex ray tracing. The new method extrapolates the wavefield into the shadow using derivatives of the travel time and the ray amplitude. We present a synthetic seismogram for a test model involving a caustic cusp.

Introduction

Hanyga and Helle (1991) developed the GRT method as an extension of the asymptotic ray theory (ART). In terms of ART seismic wavefield at point **x** may be described by:

$$\mathbf{u}(\mathbf{x},t) \sim \sum_{k=1}^{N} \mathbf{U}_k(\mathbf{x})\delta\left(t-\tau_k(\mathbf{x})\right), \tag{1}$$

where N corresponds to a number of rays passing through the receiver **x**, $\mathbf{U}_k(\mathbf{x})$ and $\tau_k(\mathbf{x})$ are corresponding ray amplitudes and traveltimes. Ordinary two-point ray tracing routines provide all mentioned above values.

ART fails at caustics because the ansatz (1) does not work there. It is possible to generalize ART getting uniformly valid asymptotic expressions utilizing an integral form of the wave field representation (oscillatory integrals in frequency domain and their time-domain analogues):

$$\mathbf{u}(\mathbf{x},t) \sim \int_{\mathbf{D}} \mathbf{a}(\mathbf{p};\mathbf{x})\delta\left(t-\Phi(\mathbf{p};\mathbf{x})\right)d\mathbf{p} \tag{2}$$

$\mathbf{a}(\mathbf{p},\mathbf{x})$ – amplitude function; $\Phi(\mathbf{p},\mathbf{x})$ – traveltime function; **p** – integration parameters. There are several asymptotic method that result into the integral formulas of the type (2): Gaussian beam summation, Chapman-Maslov method, Green-Kirchhoff's method (see references in Hanyga and Helle, 1991). Maslov theory provides rigorous theoretical basis for the expression (2). Main result of the Catastrophe is a list of typical caustics that may occur during seismic wave propagation in 3D inhomogeneous medium. For each type of caustic there is a canonical integral (FWC - function of wave catastrophe) describing the wavefield in its vicinity. And integral (2) may be always smoothly transformed into the one of canonical forms (FWC):

$$\mathbf{u}(\mathbf{x},t) \sim \int \mathbf{b}(\mathbf{q};\mathbf{c})\delta\left(t-\Psi(\mathbf{q};\boldsymbol{\alpha})\right)d\mathbf{q}, \tag{3}$$

where integration parameter **p** may be one or two dimensional; polynomials $\mathbf{b}(\mathbf{q};\mathbf{c})$ and

$\Psi(\mathbf{q};\boldsymbol{\alpha})$ are known for each caustic type; $\{\boldsymbol{\alpha}(\mathbf{x}),\mathbf{c}(\mathbf{x})\}$ denotes a set of FWC parameters. The canonical integrals (3) are well studied for all typical caustics. Thus for practical wavefield computations (receiver **x** being close to a caustic) one needs to find parameters $\{\boldsymbol{\alpha}(\mathbf{x}),\mathbf{c}(\mathbf{x})\}$ of the corresponding integral (3).

Theory

Details on the GRT theory may be found in (Hanyga and Helle, 1991). For simplicity we will further consider 1D observation geometry (*x* denotes the receiver coordinate along the profile). Computation procedure of GRT consists of the following steps:

- Find all *N* rays connecting source to the receiver at x_0.
- Standard ray tracing and dynamic ray tracing procedures provide the traveltimes $t_k(x_0)$ and the ray amplitudes $\mathbf{U}_k(x_0)$.
- 2*N* values of $\{t_k(x_0),\mathbf{U}_k(x_0)\}$ are further used to calculate 2*N* parameters $\{\boldsymbol{\alpha}(x_0),\mathbf{c}(x_0)\}$ for the canonical FWC.
- Calculate the canonical integral for particular parameters $\{\boldsymbol{\alpha}(x_0),\mathbf{c}(x_0)\}$ in order to get a synthetic seismogram at the point x_0.

Let us consider the third step of the procedure in more detail for a caustic cusp (single loop of the traveltime function, see Fig 1,a and Fig 1,b). In this case the canonical integral (3) takes the form of a single-fold integral with the following amplitude and traveltime functions

$$\begin{aligned}\Psi\left(q;\mathbf{a}(x)\right) &= t_0(x)+\alpha(x)q+\beta(x)q^2+q^4/4,\\ \mathbf{b}\left(q;\mathbf{c}(x)\right) &= \mathbf{c}_0(x)+\mathbf{c}_1(x)q+\mathbf{c}_2(x)q^2\end{aligned} \tag{4}$$

that depend on 6 parameters $\{\boldsymbol{\alpha},\mathbf{c}\}=\{t_0,\alpha,\beta,\mathbf{c}_0,\mathbf{c}_1,\mathbf{c}_2\}$. According to GRT three critical points q_i of the function $\Psi\left(q;\mathbf{a}\right)$ correspond to the three rays coalescing at a caustic cusp. Thus we get a connection between the integral formula (3) and the results of the ray tracing:

$$\Psi_{,q}\left(q_i(x);x\right)=0,\ \Psi\left(q_i(x);x\right)=t_i(x),\ \mathbf{b}\left(q_i(x);x\right)=\mathbf{U}_i(x),\ i=1,2,3. \tag{5}$$

First 2 equations in (5) form a system of nonlinear equations that may be solved for a particular receiver x_0 providing 3 critical points $q_i(x_0)$ and first three parameters $t_0(x_0)$, $\alpha(x_0)$, $\beta(x_0)$. The lust equation in (5) can be used for determining $c_0(x_0)$, $c_1(x_0)$, $c_2(x_0)$. For alight caustic zone (all *N* rays are real) the standard ray tracing and dynamic ray tracing routines provide all necessary input data for GRT. This great advantage disappears in a caustic shadow: the procedure stays unchanged but the complex ray tracing is required. This in turn implies analytic continuation of all boundaries and reflection/transmission coefficients. Thus a new routine for the complex ray tracing is required and the method becomes inefficient for complicated models with a block structure. In order to avoid complex ray tracing the standard GRT should be modified. In the vicinity of the point x_0 (all *N* rays are real) we will differentiate equations (5) with respect to the coordinate *x*:

$$\frac{d\Psi_{,q}\left(q_i(x);x\right)}{dx}=0,\ \frac{d\Psi\left(q_i(x);x\right)}{dx}=\frac{\partial t_i(x)}{\partial x},\ \frac{d\mathbf{b}\left(q_i(x);x\right)}{dx}=\frac{\partial\mathbf{U}_i(x)}{\partial x}. \tag{6}$$

Equations (6) form a set of linear equations for computing derivatives of the FWC parameters $\{\partial t_0/\partial x,\ \partial\alpha/\partial x,\ \partial\beta/\partial x,\ \partial\mathbf{c}_0/\partial x,\ \partial\mathbf{c}_1/\partial x,\ \partial\mathbf{c}_2/\partial x\}$. However the first-order derivatives $\partial t_i/\partial x$ and $\partial\mathbf{U}_i/\partial x$ should be known for all rays at point x_0. One can proceed differentiating (6) with respect to x getting the second derivatives of $\{\boldsymbol{\alpha}, \mathbf{c}\}$ at point x_0 and so on. Goldin and Duchkov (2003) developed differential equations that can be used for computation of the traveltime and the ray amplitude derivatives of arbitrary order along the ray from the source to the receiver x_0. Now it is possible to formulate the method of extrapolation for computing seismic wavefield in a caustic shadow:

- For the receiver x_0 in the alight zone of a caustic (all N rays being real) we apply GRT using values $\{t_k(x_0), \mathbf{U}_k(x_0)\}$.
- The traveltime and the ray amplitude derivatives are computed along these rays; they are used for calculating derivatives of $\{t_k(x_0), \mathbf{U}_k(x_0)\}$ with respect to x coordinate.
- The derivatives of $\{t_k(x_0), \mathbf{U}_k(x_0)\}$ are transform them into the derivatives of $\{\boldsymbol{\alpha}(x_0), \mathbf{c}(x_0)\}$ along the profile.
- Truncated Taylor series is used for extrapolating values of $\{\boldsymbol{\alpha}(x), \mathbf{c}(x)\}$ along the profile into the shadow zone (receiver x_1). Parameters $\{\boldsymbol{\alpha}(x), \mathbf{c}(x)\}$ are smooth functions of the receiver coordinate and are well predicted by the first terms of their Taylor series.
- Calculate a canonical integral for particular parameters $\{\boldsymbol{\alpha}(x_1), \mathbf{c}(x_1)\}$ in order to get a synthetic seismogram at the receiver x_1 in the caustic shadow.

Numerical example

The new method was used to compute a synthetic seismogram for a test model involving a caustic cusp. In Fig. 1,a one can see the model with two homogeneous layers. Rays of a reflected PP wave form the caustic cusp in the upper layer. A short bold line shows a receiver array: it crosses the caustic cusp. A traveltime function for this receiver array is shown in the Fig. 1,b and forms a loop. Three central receivers (triangles in Fig. 1.b) are placed in the region of multi-ray coverage (3 rays pass each receiver). The other receivers are in a caustic shadow. GRT was used to compute parameters $\{\boldsymbol{\alpha}, \mathbf{c}\}$ and a seismic waveform for 3 central receivers. For the same points we have found the derivatives of $\{\boldsymbol{\alpha}, \mathbf{c}\}$. The method of extrapolation was used to find parameters $\{\boldsymbol{\alpha}(x), \mathbf{c}(x)\}$ for the whole profile (see Fig. 2). Extrapolated values were used for computing synthetic traces in the caustic shadow. Final synthetic seismogram for the reflected PP wave is shown in Fig. 1,c. A 30 Hz Ricker wavelet pressure source was used for computations.

Conclusions

We have presented a new method of computing seismic wave field in a caustic shadow and showed its numerical application for the problem involving traveltime loop. Hanyga and Helle (1995) tested GRT and FD modeling for the same model. The comparison revealed the high degree of accuracy of GRT. The new method of extrapolation is a modification of GRT avoiding complex ray tracing in a caustic shadow. This method may be used for complicated 3D elastic models with block structure. It also shares other advantages of GRT: it is computationally cheap; it is possible to model only those waves which are of particular interest and neglect other waves which might obscure the former.

Extrapolated parameters (see Fig. 2) show that the amplitude parameters $\{\mathbf{c}_0, \mathbf{c}_1, \mathbf{c}_2\}$ can be well approximated by a constant value along the profile. Variations of $\{t_0, \alpha, \beta\}$ are well approximated by three first terms of the Taylor series. Thus we need only the first and the second traveltime derivatives for the method of extrapolation. These values are computed in course of ordinary dynamic ray tracing (no additional calculations for extrapolation).

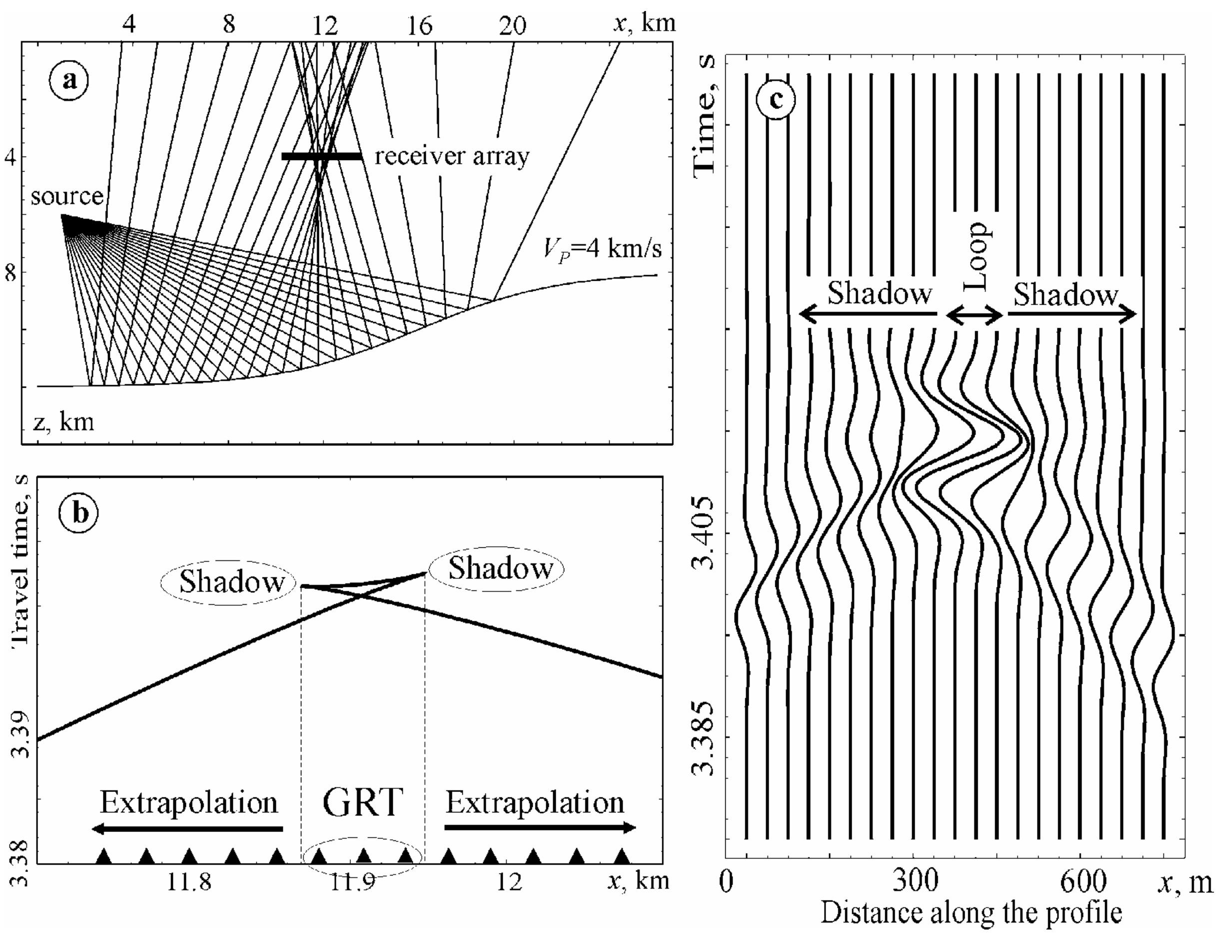

Fig. 1. Computation of synthetic seismogram; a) two-layer model, rays of the reflected PP wave, and location of the receiver array; b) traveltime loop for the receiver array in panel a; c) synthetic seismogram (see details in the text).

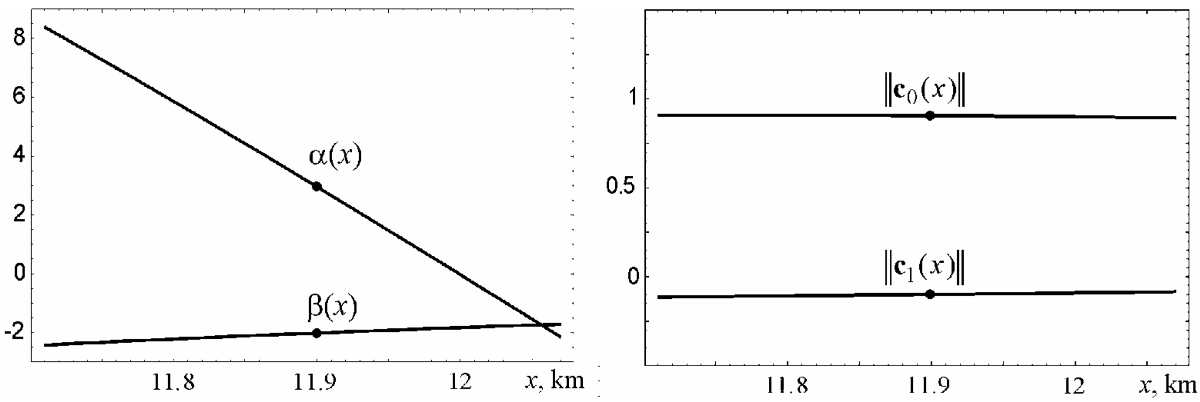

Fig. 2. Parameters $\{\boldsymbol{\alpha}(x), \mathbf{c}(x)\}$ of the canonical function (3) extrapolated from the central point over the whole receiver line.

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